Multiway Canonical Correlation Analysis for Frequency Components Recognition in SSVEP-based BCIs

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Abstract. Steady-state visual evoked potential (SSVEP)-based brain computer-interface (BCI) is one of the most popular BCI systems. An efficient SSVEP-based BCI system in shorter time with higher accuracy in recognizing SSVEP has been pursued by many studies. This paper introduces a novel multiway canonical correlation analysis (Multiway CCA) approach to recognize SSVEP. This approach is based on tensor CCA and focuses on multiway data arrays. Multiple CCAs are used to find appropriate reference signals for SSVEP recognition from different data arrays. SSVEP is then recognized by implementing multiple linear regression (MLR) between EEG and optimized reference signals. The proposed Multiway CCA is verified by comparing to the standard CCA and power spectral density analysis (PSDA). Results showed that the Multiway CCA achieved higher recognition accuracy within shorter time than that of the CCA and PSDA.

Keywords: Brain-computer interface (BCI), Canonical Correlation Analysis (CCA), Electroencephalogram (EEG), Steady-State Visual Evoked Potential (SSVEP), Tensor Decomposition.

1 Introduction

SSVEP is evoked over occipital scalp areas with the same frequency as the visual stimulus and may also include its harmonics when subject focuses on the repetitive flicker of a visual stimulus [1]. According to this mechanism, a SSVEP-based BCI can be designed to recognize the frequency components of EEG signals. In recent years, SSVEP-based BCI has been increasingly studied and has demonstrated strength including shorter calibration time and higher information transfer rate (ITR) than other types of BCIs [2]. Although SSVEP provides aforementioned advantages for BCI systems, it may be contaminated by spontaneous EEG or noise and it is still a challenge to detect it with a high accuracy, especially at a short time window (TW) [3]. Hence, how to recognize SSVEP with higher accuracy and for shorter TW is a considerably important issue for obtaining an improved SSVEP-based BCI.

A traditional and widely used method for SSVEP recognition is power spectral density analysis (PSDA). PSD is estimated from the EEG signals within a TW typically by Fast Fourier Transform (FFT), and its peak is detected to recognize the target stimulus [4]. Instead of recognizing SSVEP by directly detecting the peak of PSD, some studies also took the PSDs as features and applied linear discriminant analysis (LDA) or support vector machine (SVM) classifier to classify the target frequency [1], [5]. A TW longer than 3 s is usually required for estimating spectrum with sufficient frequency resolution when using the PSDA [4]. Such duration may limit the real-time performance of SSVEP-based BCIs. Lin et al. [6] proposed a promising and increasingly used method based on canonical correlation analysis (CCA) to recognize SSVEP. In their work, CCA was used to find the correlations between the EEG signals of multiple channels and reference signals of sine-cosine with different stimulus frequencies. Then, the target stimulus is recognized through maximizing these correlations. The use of CCA seems to provide better recognition performance than that of the PSDA since it delivers an optimization for the combination of multiple channels and improves the signal-to-noise ratio (SNR). A further comparison between the CCA and PSDA was done by Hakvoort et al. [7]. They also adopted the sine-cosine waves as reference signals used in the CCA for SSVEP recognition.

Although the CCA works quite well in SSVEP-based BCIs, we consider that the commonly used reference signals of sine-cosine may be not optimal for SSVEP recognition due to the inter-subject variability of SSVEP and effects of ongoing EEG and noises. Hence, our goal in this study is to find more efficient reference signals used in correlation analysis for SSVEP recognition. Tensor CCA proposed by Kim et al. [8] is a extension of the standard CCA and focuses on two multiway data arrays. Inspired by their work, We propose a Multiway CCA approach to discover the optimal reference signals from different modes (space and trial modes) of multidimensional EEG data and recognize SSVEP. The proposed method is verified with the EEG data of three healthy subjects and compared with the standard CCA, PSDA and the combination of PSDA and LDA (PSDA+LDA).

2 Experiment and EEG Acquisition

Three healthy volunteers (all males, aged 25, 31 and 34) participated in the experiments. The subjects were seated in a comfortable chair 50 cm from a LCD monitor (60 Hz refresh rate) in a shielded room. Four white squares, as stimuli, were flickered at four different frequencies: 8.5 Hz, 10 Hz, 12 Hz and 15Hz, respectively, on the black screen. In the experiment, each subject completed five runs with $5 \sim 10$ min rest after each of them. In each run, the subject was asked to focus on each of the four white squares for five times with a duration of 2 s for each time, respectively, with each target cue duration of one second. That is, each run contains 20 trials and totally 100 trials were completed for each subjects. EEG signals were recorded by a Biosemi Active Two amplifier at 256 Hz sampling rate ($f_s = 256$ Hz) from eight channels PO3, POz, PO4, PO7, PO8, O1, Oz and O2 placed on the standard position of the 10-20 international system. The average of them was used as reference. The EEG signals were bandpass filtered between 5 and 50 Hz.

3 Method

3.1 CCA and SSVEP Recognition

CCA is a multivariable statistical method to reveal the underlying correlation between two sets of data [9]. Consider two sets of random variables $\mathbf{X} \in \mathbb{R}^{I_1 \times J}$, $\mathbf{Y} \in \mathbb{R}^{I_2 \times J}$ and their linear combination $\tilde{\mathbf{x}} = \mathbf{w}^T\mathbf{X}$ and $\tilde{\mathbf{y}} = \mathbf{v}^T\mathbf{Y}$, CCA tries to find a pair of linear transform $\mathbf{w} \in \mathbb{R}^{I_1 \times 1}$ and $\mathbf{v} \in \mathbb{R}^{I_2 \times 1}$ to maximize the correlation between $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$, through solving the following optimization problem:

$$\rho = \max_{\mathbf{w},\mathbf{v}} \frac{E\left[\tilde{\mathbf{x}}\tilde{\mathbf{y}}\right]}{\sqrt{E\left[\tilde{\mathbf{x}}^2\right] E\left[\tilde{\mathbf{y}}^2\right]}} = \frac{\mathbf{w}^T\mathbf{X}\mathbf{Y}^T\mathbf{v}}{\sqrt{\mathbf{w}^T\mathbf{X}\mathbf{X}^T\mathbf{w}\mathbf{v}^T\mathbf{Y}\mathbf{Y}^T\mathbf{v}}}. \tag{1}$$

The maximum of ρ corresponds to the maximum canonical correlation between the canonical variates $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$.

Lin et al. [6] introduced the CCA to recognize SSVEP for the first time. Assume there are M stimulus frequencies need to be recognized. $\mathbf{X}$ consists of EEG signals from I_1 channels, and $\mathbf{Y}_m$, as a reference signals set, is constructed by sine-cosine waves at the mth stimulus frequency f_m $(m = 1, 2, \ldots, M)$:

$$\mathbf{Y}_m = \begin{pmatrix} \sin\left(2\pi f_m 1/f_s\right) & \ldots & \sin\left(2\pi f_m J/f_s\right) \\ \cos\left(2\pi f_m 1/f_s\right) & \ldots & \cos\left(2\pi f_m J/f_s\right) \\ \vdots & \vdots & \vdots \\ \sin\left(2\pi H f_m 1/f_s\right) & \ldots & \sin\left(2\pi H f_m J/f_s\right) \\ \cos\left(2\pi H f_m 1/f_s\right) & \ldots & \cos\left(2\pi H f_m J/f_s\right) \end{pmatrix}, \tag{2}$$

where H denotes the number of used harmonics (i.e., $I_2 = 2H$), J is the number of sampling points and f_s represents the sampling rate. We apply the optimization of Eq.(1) to solve the canonical correlations $\rho_1, \rho_2, \ldots, \rho_M$ corresponding to the M reference signals, respectively. Then the target stimulus frequency f_{target} is recognized as:

$$f_{\text{target}} = \max_{f_m} \rho_m, \quad m = 1, 2, \ldots, M. \tag{3}$$

Although the CCA works quite well for SSVEP recognition, sine-cosine waves may be not the optimal reference signals in using correlation analysis since they do not contain any information about the inter-subject variability and trial-to-trial variability. We consider that the recognition accuracy may be further improved by optimizing the reference signals. We will show how to find more efficient reference signals by a novel Multiway CCA approach from experimental multidimensional EEG data in the next section.

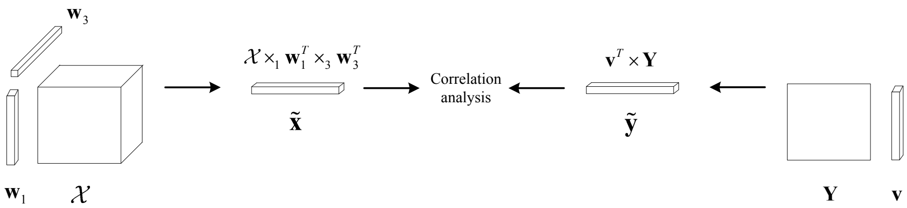

Fig. 1. Illustration of Multiway CCA mode.

3.2 Multiway CCA and SSVEP Recognition

A tensor is a multiway array of data and the order of the tensor is the number of dimensions, also known as ways or models [10]. A first-order tensor is a vector and a second-order tensor is a matrix. A Nth-order tensor is denoted by $\mathcal{X} = (\mathcal{X})_{i_1 i_2 \ldots i_N} \in \mathbb{R}^{I_1 \times I_2 \times \ldots \times I_N}$. The n-mode product of the tensor with a vector $\mathbf{w} \in \mathbb{R}^{I_n \times 1}$ is

$$\left(\mathcal{X} \times_n \mathbf{w}^T\right)_{i_1 \ldots i_{n-1} i_{n+1} \ldots i_N} = \sum_{i_n=1}^{I_n} x_{i_1 i_2 \ldots i_N} w_{i_n}. \tag{4}$$

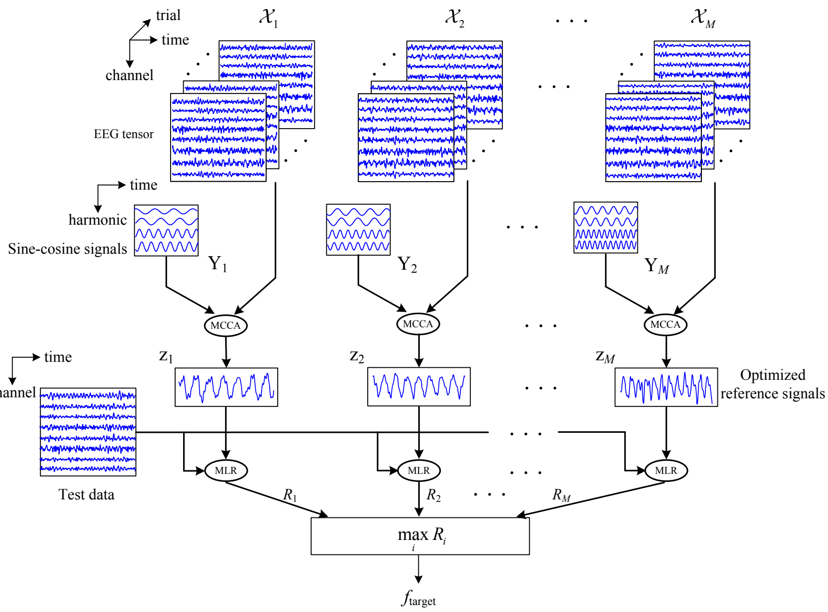

Fig. 2. Illustration of Multiway CCA approach for SSVEP recognition. Here, MCCA denotes the Multiway CCA.

Multiway CCA mode. Tensor CCA is an extension of the standard CCA, which focuses on inspecting the correlation between two multiway data arrays, instead of two sets of vector-based variables [8]. Drawing on the idea of Tensor CCA, we introduce a Multiway CCA which maximizes the correlation between multiway data (tensor) and two-way data (matrix) to optimize the reference signals used in correlation analysis for SSVEP recognition. We consider that EEG data from the trials with a specific stimulus frequency form a third-order (three-way) data tensor $\mathcal{X} \in \mathbb{R}^{I \times J \times K}$ (channel $\times$ time $\times$ trial) and a original reference signal matrix $\mathbf{Y} \in \mathbb{R}^{2H \times J}$ (harmonic $\times$ time) is constructed by the sine-cosine signals with frequencies as the stimulus frequency and its higher harmonics. Our aim is to find more efficient reference signals for SSVEP recognition from time domain (i.e. optimizing the channels and trials data ways) based on the original reference signals of sine-cosine. Then, the canonical correlation between mode-2 of $\mathcal{X}$ and $\mathbf{Y}$ is considered. The proposed Multiway CCA finds linear transforms $\mathbf{w}_1 \in \mathbb{R}^{I \times 1}$, $\mathbf{w}_3 \in \mathbb{R}^{K \times 1}$ and $\mathbf{v} \in \mathbb{R}^{2H \times 1}$ such that

$$\rho = \max_{\mathbf{w}_1, \mathbf{w}_3, \mathbf{v}} \frac{E\left[\tilde{\mathbf{x}}\tilde{\mathbf{y}}\right]}{\sqrt{E\left[\tilde{\mathbf{x}}^2\right] E\left[\tilde{\mathbf{y}}^2\right]}} \tag{5}$$

is maximized, where $\tilde{\mathbf{x}} = \mathcal{X} \times_1 \mathbf{w}_1^T \times_3 \mathbf{w}_3^T$ and $\tilde{\mathbf{y}} = \mathbf{v}^T \times \mathbf{Y}$. Fig. 1 illustrates the Multiway CCA mode. For solving this problem, we adopt an alternating algorithm which fixes $\mathbf{w}_3$ to solve $\mathbf{w}_1$ and $\mathbf{v}$, then fixes $\mathbf{w}_1$ and $\mathbf{v}$ to solve $\mathbf{w}_3$, and repeats this procedure until convergence criterion is satisfied. Then, the optimal reference signal denoted by $\mathbf{z} \in \mathbb{R}^{1 \times J}$ can be obtained by:

$$\mathbf{z} = \mathcal{X} \times_1 \mathbf{w}_1^T \times_3 \mathbf{w}_3^T. \tag{6}$$

When compared with the standard sine-cosine signals, the optimized reference signal contains not only the ideal SSVEP frequency components but also the information of inter-subject variability and trial-to-trial variability.

Multiway CCA based SSVEP recognition. We represent the experimental EEG data corresponding to the mth stimulus frequency f_m $(m = 1, 2, \ldots, M)$ as a third-order tensor $\mathcal{X}_m \in \mathbb{R}^{I \times J \times K}$ (channel$\times$time$\times$trial). The Multiway CCA is implemented to maximize the correlation between the EEG data tensor and the corresponding sine-cosine signals $\mathbf{Y}_m \in \mathbb{R}^{2H \times J}$ (harmonic$\times$time) defined by Eq.(2), and find the optimal reference signals denoted by $\mathbf{z}_m \in \mathbb{R}^{1 \times J}$, which will be used to replace the original sine-cosine based reference signals. Then, with the new reference signals, multiple linear regression (MLR) [12], a correlation technique focusing on a variable and a set of variables, is utilized to recognize target stimulus frequency. We consider the relationship between the test EEG data $\mathbf{X}_{\text{test}} \in \mathbb{R}^{I \times J}$ and optimized reference signal $\mathbf{z}_m$ as a multiple regression model, i.e.,:

$$\mathbf{z}_m = \mathbf{b}_m^T \mathbf{X}_{\text{test}} + \mathbf{e}_m, \tag{7}$$

where $\mathbf{b}_m \in \mathbb{R}^{I \times 1}$ is a coefficient vector to be estimated and $\mathbf{e}_m \in \mathbb{R}^{1 \times J}$ is a noise vector with zero mean and constant variance. With least square method,

Algorithm 1: Multiway CCA algorithm for SSVEP Recognition

Input: M EEG tensor data $\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_M \in \mathbb{R}^{I \times J \times K}$ and sine-cosine signals $\mathbf{Y}_1, \mathbf{Y}_2, \ldots, \mathbf{Y}_M \in \mathbb{R}^{2H \times J}$ corresponding to M stimulus frequencies, respectively. A test EEG data $\mathbf{X}_{\text{test}} \in \mathbb{R}^{I \times J}$.
Output: Recognition result f_{target}.
for $m = 1$ *to* M **do**
  Random initialization for $\mathbf{w}_{m,3}$ and do $\tilde{\mathbf{X}}_m \leftarrow \mathcal{X}_m \times_3 \mathbf{w}_{m,3}^T$.
  repeat
    Find $\mathbf{w}_{m,1}$, $\mathbf{v}_m$ which maximize the correlation between $\tilde{\mathbf{X}}_m$ and $\mathbf{Y}_m$ by the CCA. Do $\tilde{\mathbf{X}}_m \leftarrow \mathcal{X}_m \times_1 \mathbf{w}_{m,1}^T$, $\tilde{\mathbf{y}}_m \leftarrow \mathbf{v}_m^T \times \mathbf{Y}_m$.
    Find $\mathbf{w}_{m,3}$ which maximizes the correlation between $\tilde{\mathbf{X}}_m$ and $\tilde{\mathbf{y}}_m$ by the CCA. Do $\tilde{\mathbf{X}}_m \leftarrow \mathcal{X}_m \times_3 \mathbf{w}_{m,3}^T$.
  until *the maximum number of iterations is reached* ;
  Compute the optimized reference signal $\mathbf{z}_m \leftarrow \mathcal{X}_m \times_1 \mathbf{w}_{m,1}^T \times_3 \mathbf{w}_{m,3}^T$.
end
for $m = 1$ *to* M **do**
  Implement MLR between $\mathbf{X}_{\text{test}}$ and $\mathbf{z}_m$ to obtain the correlation R_m.
end
Recognize target stimulus frequency as $f_{\text{target}} = \max_{f_m} R_m, (m = 1, 2, \ldots, M)$.

the esimation of $\mathbf{b}_m$ is solved as:

$$\hat{\mathbf{b}}_m = \left(\mathbf{X}_{\text{test}} \mathbf{X}_{\text{test}}^T\right)^{-1} \mathbf{X}_{\text{test}} \mathbf{z}_m^T, \tag{8}$$

and the estimated vector of fitting values $\hat{\mathbf{z}}_m$ is computed as:

$$\hat{\mathbf{z}}_m = \hat{\mathbf{b}}_m^T \mathbf{X}_{\text{test}} = \mathbf{z}_m \mathbf{X}_{\text{test}}^T \left(\mathbf{X}_{\text{test}} \mathbf{X}_{\text{test}}^T\right)^{-1} \mathbf{X}_{\text{test}}, \tag{9}$$

Then, the correlation coefficient R_m which reflects the relationship between $\mathbf{X}_{\text{test}}$ and $\mathbf{z}_m$ is calculated as:

$$R_m = \sqrt{1 - \frac{\|\mathbf{z}_m - \hat{\mathbf{z}}_m\|_2^2}{\|\mathbf{z}_m - E\left[\mathbf{z}_m\right]\|_2^2}}, \tag{10}$$

where $\|\cdot\|_2$ denotes l_2-norm. Larger R_m implies more significant relationship between $\mathbf{X}_{\text{test}}$ and $\mathbf{z}_m$. Then, the target stimulus frequency is recognized as:

$$f_{\text{target}} = \max_{f_m} R_m, (m = 1, 2, \ldots, M). \tag{11}$$

The algorithm of the proposed Multiway CCA for SSVEP recognition is summarized in Algorithm 1. Fig. 2 illustrates SSVEP recognition based on the Multiway CCA. For each subject, five-fold cross-validation is used to estimated average classification accuracy. More specifically, a procedure, in which the EEG data from four runs (80 trials) are used to optimize the reference signals and that from the left-out run (20 trials) is used for SSVEP recognition, is repeated five times so that each run served once for SSVEP recognition validation.

4 Results

The proposed Multiway CCA was compared with the standard CCA, PSDA and PSDA+LDA. EEG data from all eight channels were used as the inputs for the standard CCA and Multiway CCA. For the PSDA, PSDs were estimated by $4f_s$-point-FFT (i.e., the frequency resolution is 0.25 Hz) from the EEG data with a bandwidth of 0.5 Hz, averaged on the channels O1, Oz and O2. For the PSDA+LDA, we took the PSDs as features and applied a 4-class classifier built by combining six single LDAs to classify the target frequency. The average accuracy was also estimated by a five-fold cross-validation. Fig. 3 shows the recognition accuracy of the four methods for different subjects and harmonic combinations. For different subjects and harmonic combinations, while the standard CCA performed better than the PSDA and PSDA+LDA, the proposed Multiway CCA yielded higher recognition accuracies than the standard CCA for most time window (TW) lengths. There was no big difference between the accuracy of the PSDA and PSDA+LDA. For most of the four methods, the performance in using more harmonics was slightly better than that in using fewer harmonics.

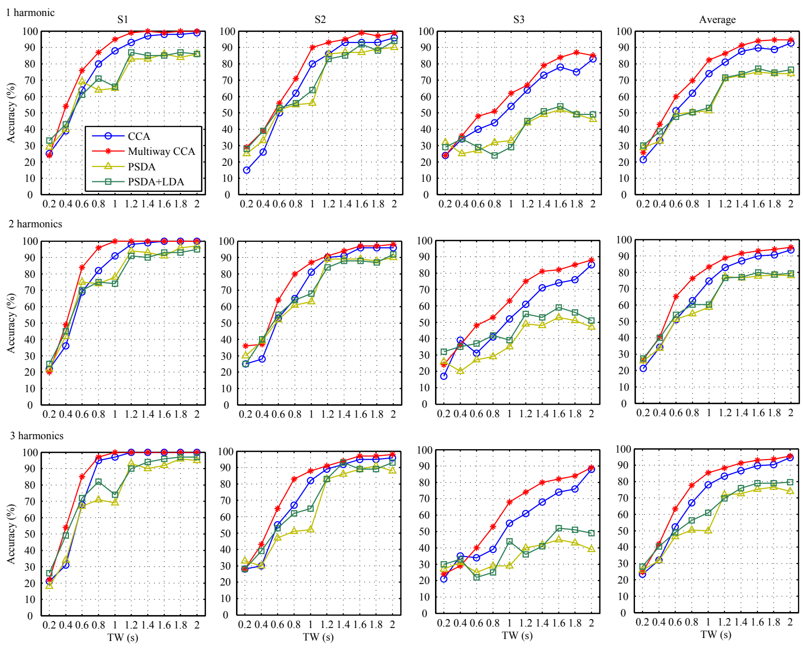

Fig. 3. SSVEP recognition accuracies obtained by the standard CCA, Multiway CCA, PSDA and the combination of PSDA and LDA, across different time window (TW) lengths, in using different harmonic combinations, for three subjects. The rightmost column shows the accuracies averaged on all subjects. 1 harmonic: fundamental frequency only, 2 harmonics: fundamental frequency and second harmonic, 3 harmonics: fundamental frequency, second and third harmonics.

Information transfer rate (ITR) [11] was also used to evaluate the performance of the CCA and Multiway CCA further. The ITR can be computed as:

$$B = \log_2 N + Acc \times \log_2 Acc + (1 - Acc) \times \log_2 \left[(1 - Acc)/(N - 1)\right], \quad (12)$$

$$\mathrm{ITR} = B \times 60/T, \quad (13)$$

where the bit rate or bits/trial is denoted by B, N is the number of stimulus frequency, Acc is the recognition accuracy and T represents the duration per trial. Table 1 presents the recognition accuracies and ITRs of the CCA and Multiway CCA in using different channel combinations and TW lengths. Here, we focus on the TW lengths in range of 0.8 s $\sim$ 1.4 s to compromise between recognition accuracy and speed. For all of the four TW lengths, the Multiway CCA yielded higher recognition accuracies and higher ITRs than that of the CCA for various channel combinations. Furthermore, if fewer number of channels were used, bigger advantages over the CCA seemed to be achieved by the Multiway CCA. For both methods, the combination of more channels used yielded better performance than that in using fewer channels.

Table 1. Accuracy (Acc) (%) and information transfer rate (ITR) (bits/min) of the standard CCA and Multiway CCA (MCCA) in using different channel combinations and within different time window lengths (TW) (s), averaged on all subjects.

TW	Channel	CCA		MCCA	
		Acc	ITR	Acc	ITR
0.8					
	8 channels	67.0	25.2	77.7	34.3
	6 channels	68.0	25.7	74.3	32.4
	3 channels	60.7	17.8	70.7	27.8
1.0					
	8 channels	78.0	30.9	85.3	38.7
	6 channels	73.7	29.8	81.3	36.3
	3 channels	67.7	22.9	78.0	32.6
1.2					
	8 channels	83.3	34.0	88.3	38.0
	6 channels	80.3	32.0	85.7	37.0
	3 channels	72.7	26.6	82.3	33.2
1.4					
	8 channels	86.7	33.8	91.3	37.8
	6 channels	85.0	33.3	88.0	35.4
	3 channels	78.7	28.9	83.3	33.1

Note: 8 channels: PO3, POz, PO4, PO7, PO8, O1, Oz and O2. 6 channels: PO3, POz, PO4, O1, Oz, O2. 3 channels: O1, Oz, O2.

5 Discussion and Conclusion

In this study, a Multiway CCA approach was proposed to recognize the stimulus frequency for SSVEP-based BCI. In this method, multiple CCAs were implemented between the EEG tensor data and sine-cosine signals to find appropriate reference signals used in correlation analysis for SSVEP recognition. After that, multiple linear regression was applied to inspect the correlation between the test EEG data and optimized reference signals for SSVEP recognition. From the results, the Multiway CCA achieved higher accuracy than that of the standard CCA, PSDA and the combination of PSDA and LDA, within shorter TW length. This shows the proposed method is promising for enhancing the real-time performance of SSVEP-based BCIs. Also, the better performance of the Multiway CCA confirmed that the reference signals optimized from space and trial data modes were more efficient than the commonly used sine-cosine signals for SSVEP recognition, since they might contain some information of subject-specific and trial-to-trial variability. It is possible to develop an online learning algorithm which gives real-time updates to the reference signals so that an adaptive SSVEP-based BCI can be established, which will be our future study.

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