Blind Equalization via Approximate Maximum Likelihood Source Separation

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Abstract

Blind equalization of single input multiple output (SIMO) FIR channels can be reformulated as the problem of blind source separation. It was shown in [4] that the natural gradient-based source separation method could recover source successfully for ill-conditioned channels since it has the equivariant property. However, the effect of additive noise was not considered in [4]. In this letter we develop a new approximate maximum likelihood source separation (AMLSS) method using the natural gradient and apply it to the task of blind equalization. We show that the proposed method outperforms the BSBE [4] in the presence of Gaussian noise.

Indexing terms: Blind equalization, blind source separation, maximum likelihood, natural gradient.

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1 Introduction

Blind equalization is an important problem in digital communications and signal processing. In the context of blind equalization of SIMO channels, M-dimensional observation vector $\boldsymbol{x}(k)$ is assumed to be generated from an unknown source signal $s(k)$ through M different FIR filters, i.e.,

$$\boldsymbol{x}(k) = \sum_{p=0}^{L} \boldsymbol{h}(p) s(k-p) + \boldsymbol{v}(k), \tag{1}$$

where $\{\boldsymbol{h}(p)\}$ are the impulse responses of channels with degree L and $\boldsymbol{v}(k)$ is the additive white Gaussian noise that is assumed to be statistically independent of the source signal $s(k)$.

Stacking N successive samples of the observation vector, i.e.,

$$\mathcal{X}(k) = [\boldsymbol{x}^T(k), \ldots, \boldsymbol{x}^T(k-N+1)]^T, \tag{2}$$

the model (1) can be reformulated as

$$\mathcal{X}(k) = \mathcal{H}_N \boldsymbol{s}(k) + \mathcal{V}(k), \tag{3}$$

where

$$\mathcal{H}_N = \begin{bmatrix} \boldsymbol{h}(0) & \boldsymbol{h}(1) & \cdots & \boldsymbol{h}(L) & 0 & \cdots & \cdots & 0 \\ 0 & \boldsymbol{h}(0) & \boldsymbol{h}(1) & \cdots & \boldsymbol{h}(L) & 0 & \cdots & 0 \\ \vdots & & & & & & & \vdots \\ 0 & \cdots & \cdots & 0 & \boldsymbol{h}(0) & \boldsymbol{h}(1) & \cdots & \boldsymbol{h}(L) \end{bmatrix}, \tag{4}$$

and

$$\boldsymbol{s}(k) = [s(k), \ldots, s(k-N-L+1)]^T \tag{5}$$

$$\mathcal{V}(k) = [\boldsymbol{v}^T(k), \ldots, \boldsymbol{v}^T(k-N+1)]^T. \tag{6}$$

Here we assume that the channels are coprime and $N \geq \frac{L}{M-1}$. Then the matrix $\mathcal{H}_N$ is a tall matrix with full column rank. Since Eq. (3) is a linear data model, the task of blind equalization can be solved by blind source separation (BSS) or independent component analysis (ICA). It was pointed out in [4] that the natural gradient-based BSS method could recover source signal successfully even when some of zeros of channels are close, whereas second-order based blind identification methods (for example see [3] and references therein) had difficulty. However, the method in [4] did not take the effect of additive noise into account. In this letter we develop the AMLSS method and show that it can recover source successfully for ill-conditioned channels in the presence of white Gaussian noise.

2 The New Method

In the data model (3), the likelihood function is given by

$$p(\mathcal{X}|\mathcal{H}_N) = \int \exp\left\{-\mathcal{E}(\boldsymbol{s})\right\} d\boldsymbol{s}, \tag{7}$$

where $\mathcal{E}(\boldsymbol{s})$ is the energy function that is defined by

$$\mathcal{E}(\boldsymbol{s}) = -\log p(\mathcal{X}|\mathcal{H}_N, \boldsymbol{s}) - \log r(\boldsymbol{s}), \tag{8}$$

where $\log r(\boldsymbol{s})$ is the non-Gaussian log-prior for source.

We assume that $\mathcal{E}(\boldsymbol{s})$ has a local quadratic form around a most probable value of $\boldsymbol{s}$, $\widehat{\boldsymbol{s}}$. We also assume that the noise $\boldsymbol{v}(k)$ is an isotropic Gaussian process with zero mean and variance σ^2. Then the Laplace approximation leads to the log-likelihood that has the form

$$\log p(\mathcal{X}|\mathcal{H}_N) = -\frac{1}{2\sigma^2}\left\|\mathcal{X} - \mathcal{H}_N\widehat{\boldsymbol{s}}\right\|^2 + \log r(\widehat{\boldsymbol{s}}) - \frac{1}{2}\log\det\left(\nabla^2\mathcal{E}(\widehat{\boldsymbol{s}})\right) + C, \tag{9}$$

where C is some constant which does not depend on $\mathcal{H}_N$ and

$$\nabla^2\mathcal{E}(\widehat{\boldsymbol{s}}) = \frac{\mathcal{H}_N^T\mathcal{H}_N}{\sigma^2} - \nabla^2\log r(\widehat{\boldsymbol{s}}). \tag{10}$$

The most probable value of $\boldsymbol{s}$ is inferred by LS estimate, i,e.,

$$\widehat{\boldsymbol{s}} = \mathcal{H}_N^{\#}\mathcal{X}, \tag{11}$$

where $\mathcal{H}_N^{\#}$ is the pseudo-inverse of $\mathcal{H}_N$. We assume that the variance of noise, σ^2 is small, i.e.,

$$\nabla^2\mathcal{E}(\boldsymbol{s}) \approx \frac{\mathcal{H}_N^T\mathcal{H}_N}{\sigma^2}. \tag{12}$$

With these, the gradient of the log-likelihood (9) is given by

$$\nabla\log p(\mathcal{X}|\mathcal{H}_N) = -\mathcal{H}_N\left(\mathcal{H}_N^T\mathcal{H}_N\right)^{-1}\left\{\boldsymbol{I} - \varphi(\widehat{\boldsymbol{s}})\widehat{\boldsymbol{s}}^T\right\}, \tag{13}$$

where $\varphi(\cdot)$ is the elementwise negative score function.

Motivated from the result in [5], we calculate the natural gradient (denoted by $\widetilde{\nabla}$) of the log-likelihood (9) using the invariance of the Riemannian metric in the Lie group [1]. It has the form

$$\widetilde{\nabla} \log p(\mathcal{X}|\mathcal{H}_N) = \left\{\mathcal{H}_N \mathcal{H}_N^T + \mathcal{P}\right\} \nabla \log p(\mathcal{X}|\mathcal{H}_N), \tag{14}$$

where $\mathcal{P}$ is the projection matrix which is eventually chosen such that $\mathcal{P}\nabla \log p(\mathcal{X}|\mathcal{H}_N) = 0$ to minimize the noise effect as in [5].

Algorithm Outline: AMLSS

- Given the current estimate of the mixing matrix, $\widehat{\mathcal{H}_N}(k)$, we infer the source vector by

$$\widehat{\boldsymbol{s}}(k) = \left(\widehat{\mathcal{H}_N}^T(k)\widehat{\mathcal{H}_N}(k)\right)^{-1} \widehat{\mathcal{H}_N}^T(k)\boldsymbol{x}(k). \tag{15}$$

- Using $\widehat{\boldsymbol{s}}(k)$ and $\widehat{\mathcal{H}_N}(k)$, we find the new estimate of the mixing matrix, $\widehat{\mathcal{H}_N}(k+1)$ by

$$\widehat{\mathcal{H}_N}(k+1) = \widehat{\mathcal{H}_N}(k) - \eta\widehat{\mathcal{H}_N}(k)\left\{\boldsymbol{I} - \varphi(\widehat{\boldsymbol{s}}(k))\widehat{\boldsymbol{s}}^T(k)\right\}. \tag{16}$$

- These two steps are repeated until $\widehat{\mathcal{H}_N}$ converges.

3 Numerical Example

We compare the performance of our proposed method and the BSBE in [4]. In the numerical experiment, we used two channels with the degree $L = 3$, $\boldsymbol{h}(0) = [.378,\ .379]^T$, $\boldsymbol{h}(1) = [-.7559,\ -.7543]^T$, $\boldsymbol{h}(2) = [.378,\ .379]^T$, and $\boldsymbol{h}(3) = [.378,\ .379]^T$. A BPSK modulation was considered, so source $s(k)$ is a binary-distributed ($\{\pm 1\}$) signal. At each SNR, 10 different runs were carried out with different realization of white Gaussian noise and the average of bit error rate (BER) was computed. In both BSBE and our method, the learning rate $\eta = .001$ was used and the cubic nonlinear function $\varphi_i(\widehat{s}) = |\widehat{s}|^2\widehat{s}$. Since the proposed method takes the additive white Gaussian noise into account, it outperformed the BSBE (see Fig. 1).

4 Conclusion

We presented a method of AMLSS in the presence of Gaussian noise and applied it to the task of blind equalization of SIMO channels successfully. We demonstrated that the useful behavior of the AMLSS-based blind equalization for ill-conditioned channels and in the presence of Gaussian noise. The algorithm (16) is similar to the one in [2], but here we derived it in the framework of maximum likelihood.

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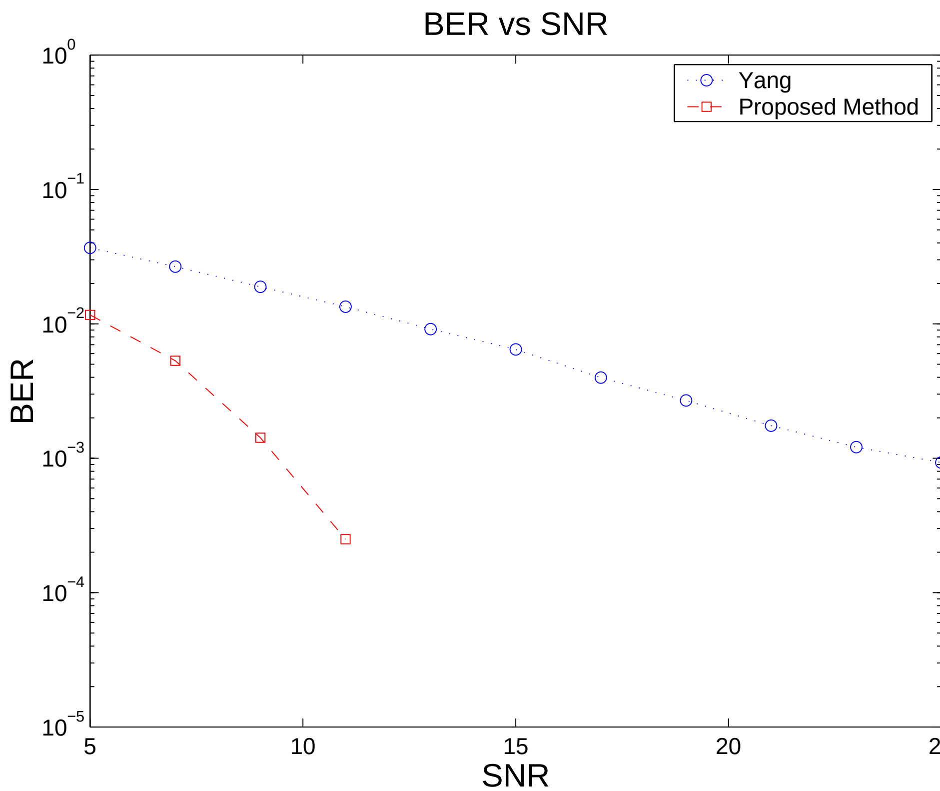

Figure 1: Performance comparison of the proposed method and the BSBE [4].